DualDegree Admission Test (Dept. of CSE) IIT Guwahati

Time: 09:00 - 10:00 (1 Hr) **Date: June 3, 2024**

Question & Answer Booklet: (9 Questions, 10 Marks each)

Name:		**App. No.:**	
Signature:		**Category:** (Gen/OBC/ SC/ST/PwD)	

Instructions

1. This question set has three sections namely **Mathematical Foundations**; **Programming, Data Structure and Algorithms**; and **Computer Systems**.
2. You can attempt any question from any section. However, you have to write which **SIX** answers we should evaluate. You **Must** mention those questions numbers below in the provided box.
3. Ensure that there are 4 printed sheets.
4. Answers should be written **only** within the space provided. Answers written outside the provided space **will not** be evaluated.
5. **No clarifications** on questions will be entertained. State your assumptions, if any.
6. Ask for separate sheets for **Rough Work**. **Do Not** do any rough work in this booklet.
7. Write your **name** and **application number** on the extra sheets also, if any taken for rough work, and return the sheets after the exam.
8. There is **NO negative marks** for incorrect answer of any question.

Answer any SIX questions and put their question numbers

Answered question #							Total
Marks (Office use only)							

Section 1: Mathematical Foundations

Q1. (a) Given the three planes $u + v + w + z = 6$, $u + w + z = 4$, and $u + w = 2$, all in four-dimensional space, determine whether their intersection is a line or a point or an empty set. [2]

(b) For which value(s) of q (if any) is the following system consistent ? [2]

$$x + 4y + 3z = 1,$$
$$q^3x + 4q^3y + 3q^3z = 64q.$$

(c) For what value(s) of x, if any, does the following matrix has at least one repeated eigenvalue. [4]

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & x & 2 \\ 0 & x & 2 \end{pmatrix}$$

(d) Division by zero during forward elimination steps in Naive Gaussian elimination of the set of equation $AX = B$ implies the coefficient matrix A (i) is invertible (ii) is nonsingular (iii) is singular (iv) may be singular or nonsigular
Give Justification. No partial marks. Marks will be given only when the justification is correct. [2]

Q2. (a) Find a closed form expression for the generating function of the sequence $\{a_n\}$, where, $a_n = 2n + 3 \ \ \forall n = 0, 1, 2, \cdots$ [3]

(b) Three friends usually go for coffee together and decide to make payment by the following rule: each person flips a coin and the person whose flip does not match with the other two, will pay. For example, if three flips are *head*, *head* and *tail*, the person with *tail* flip will pay. If all three flips are the same, then they make another round and continue until there is an odd person. Find out the probability that exactly three rounds of flips are made. [3]

(c) A communication system consists of n components, each of which will independently function with probability p. The total system will be able to operate if at least one-half of its components function. For what values of p is a 5-component system more likely to operate effectively than a 3-component system? [4]

Q3. (a) Let X be a set and let R be the relation defined on $P(X)$ as follows

$$(A, B) \in R \leftrightarrow A \cap B \neq \emptyset$$

Is R : (i) reflexive ? (ii) symmetric? (iii) transitive?
(Marks will be given only after correct justification) [1+1+1]

(b) If r is irrational number then justify whether $r^{\frac{1}{5}}$ is rational or irrational [3]

(c) Without using binomial expansion show that $\sum_{i=0}^{m} \binom{n+i}{i} = \binom{n+m+1}{m}$ [4]

Section 2: Programming, Data Structure and Algorithms

Q4. There are four people who want to cross a bridge at night. They have only one torch and the bridge is only wide enough to allow two people to cross at a time. It takes each person a different amount of time to cross the bridge (given in minutes): A takes 1 minute, B takes 2 minutes, C takes 5 minutes, and D takes 10 minutes. When two people cross the bridge together, they must move at the slower person's pace. Carrying a torch is a must while crossing the bridge. What is the minimum amount of time needed for all four people to cross the bridge? [10]

Q5. Consider the weighted undirected simple complete graph K_n with n vertices and $\frac{n(n-1)}{2}$ edges. Vertices are indexed from 1 to n. The Weight of each edge connecting vertices i and j is $i + j$. We are building the minimum spanning tree for this graph using the Kruskal's Algorithm. We are using disjoint set forest to represent components. We are using union by size heuristic while merging the components. Suppose that we have inserted m edges in the minimum spanning tree ($0 < m < n$). How many components will be there? For each component, list the vertices. [10]

Q6. Using decision tree model depict working of insertion sort on any input of size three. [10]

Q7. Consider the following algorithm for sorting a given set of integers. Is the algorithms correct? If your answer is no then, provide an input on which the algorithm will fail to sort the input correctly. If your answer is yes, prove the correctness of the algorithm. [10]

```
/* a[0] to a[n-1] is the array to sort */
void sortAlgo (int[] a, int n)
{
    int i, j, k, h, v;
    int[] cols = {1391376, 463792, 198768, 86961, 33936,
                  13776, 4592, 1968, 861, 336, 112, 48,
                   21, 7, 3, 1}
    for (k=0; k<16; k++)
    {
        h=cols[k];
        for (i=h; i<n; i++)
        {
            v=a[i];
            j=i;
            while (j>=h && a[j-h]>v)
            {
                a[j]=a[j-h];
                j=j-h;
            }
            a[j]=v;
        }
    }
}
```

Section 3: Computer Systems

Q8. (a) Draw a neat, labeled diagram of a 4:1 multiplexer and 4:2 encoder. [6]

(b) Let R and S be two 4-bit registers that stores number in 2'complement form. Let R and S contain the binary values 1010 and 0101, respectively. What are the numbers stored in R and S (in decimal)? [4]

Q9. (a) Draw a neat, labeled process state diagram. Clearly mark the process state and transitions. [4]

(b) Which layers of OSI reference model handle the following functions? (i) congestion control (ii) medium access (iii) signal encoding (iv) crash recovery (v) segmentation and reassembly (vi) data encryption [6]